Park Hill Thorns Federation

Routes through Calculations

January 2019

The policy is based on the following ideas:

- Children should develop a clear conceptual understanding of arithmetic processes.
- Children should have fluent recall of number facts, including addition facts within 10, single digit sums and times table facts.
- Children should develop a secure understanding of efficient written algorithms for arithmetic by Year 4.
- Related operations (e.g. addition and subtraction) should be introduced together, with explicit links between them.
- The role of place value in arithmetic processes should be modelled and made clear from the start of Y2.

Overview

	Reception / Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Addition	Combining two parts to make a whole: part whole model. Starting at the bigger number and counting on- using cubes. Exchanging to make 10 using ten frame.	Adding three single digits. Column layout – no exchanging.	Column layout- exchanging. Using place value counters (up to 3 digits).	Column layout with exchanging. (up to 4 digits)	Column layout with exchanging. Add decimals with up to two decimal places	Column layout- exchanging. Abstract methods.
Subtraction	Taking away ones Counting back Find the difference Part whole model Make 10 using the ten frame	Counting back Find the difference Part whole model Make 10 Column layout - no exchanging	Column layout with exchanging (up to 3 digits using place value counters)	Column layout with exchanging. (up to 4 digits)	Column layout with exchanging. Subtract decimals with up to two decimal places	Column layout with exchanging. Abstract methods.
Multiplication	Recognising and making equal groups. Doubling Counting in multiples Use cubes, Numicon and other objects in the classroom	Arrays- showing commutative multiplication	Column Short multiplication (2 digit number multiplied by 1 digit)	Column short multiplication (2 and 3 digit multiplied by 1 digit)	Column long multiplication (up to 4 digit numbers multiplied by 1 or 2 digits)	Column Long multiplication (multi digit up to 4 digits by a 2-digit number)
Division	Sharing objects into groups Division as grouping e.g. I have 12 sweets and put them in groups of 3, how many groups? Use cubes and draw round 3 cubes at a time.	Division as grouping Division within arrays- linking to multiplication Repeated subtraction	Division with a remainder-using lollipop sticks, times tables facts and repeated subtraction. 2d divided by 1d using base 10 or place value counters	Short division (up to 3 digits by 1 digit- concrete and pictorial)	Short division (up to 4 digits by a 1 digit number including remainders)	Long division with place value counters (up to 4 digits by a 2 digit number) Children should exchange into the tenths and hundredths column too

Fluency in number facts

U + U facts

+	0	1	2	3	4	5	6	7	8	9	10
0	0 + 0	0 + 1	0 + 2	0 + 3	0 + 4	0 + 5	0 + 6	0 + 7	0 + 8	0 + 9	0 + 10
1	1 + 0	1 + 1	1 + 2	1 + 3	1 + 4	1 + 5	1 + 6	1 + 7	1 + 8	1 + 9	1 + 10
2	2 + 0	2 + 1	2 + 2	2 + 3	2 + 4	2 + 5	2 + 6	2 + 7	2 + 8	2 + 9	2 + 10
3	3 + 0	3 + 1	3 + 2	3 + 3	3 + 4	3 + 5	3 + 6	3 + 7	3 + 8	3 + 9	3 + 10
4	4 + 0	4 + 1	4 + 2	4 + 3	4 + 4	4 + 5	4 + 6	4 + 7	4 + 8	4 + 9	4 + 10
5	5 + 0	5 + 1	5 + 2	5 + 3	5 + 4	5 + 5	5 + 6	5 + 7	5 + 8	5 + 9	5 + 10
6	6 + 0	6 + 1	6 + 2	6 + 3	6 + 4	6 + 5	6 + 6	6 + 7	6 + 8	6 + 9	6 + 10
7	7 + 0	7 + 1	7 + 2	7 + 3	7 + 4	7 + 5	7 + 6	7 + 7	7 + 8	7 + 9	7 + 10
8	8 + 0	8 + 1	8 + 2	8 + 3	8 + 4	8 + 5	8 + 6	8 + 7	8 + 8	8 + 9	8 + 10
9	9 + 0	9 + 1	9 + 2	9 + 3	9 + 4	9 + 5	9 + 6	9 + 7	9 + 8	9 + 9	9 + 10
10	10 + 0	10 + 1	10 + 2	10 + 3	10 + 4	10 + 5	10 + 6	10 + 7	10 + 8	10 + 9	10 + 10

- Adding 1
- Adding 2
- Bonds to 10
- Adding 0
- Adding 10
- Doubles
- Bridging/ compensating
- Near doubles

Group A: Year 1 (Within 10)	Alongside
Adding 1 (e.g. 7 + 1 and 1 + 7)	
Doubles of numbers to 5 (e.g. 4 + 4)	
Adding 2 (e.g. 4 + 2 and 2 + 4)	Partitioning 2, 3, 4, 5, 6 and 10
Number bonds to 10 (e.g. 8 + 2 and 2 + 8)	
Adding 10 to a number (e.g. 5 + 10 and 10 + 5)	
Adding 0 to a number (e.g. 3 + 0 and 0 + 3)	
Near doubles (e.g. 3 + 4 and 4 + 3)	Partitioning 7, 8 and 9
The ones without a family! 5 + 3, 3 + 5, 6 + 3, 3 + 6	
Group B: Year 2 (Bridging 10)	
Doubles of numbers to 10 (e.g. 7 + 7)	
Near doubles (e.g. 5 + 6 and 6 +5)	Partitioning 11 – 20 into single digit addends
Bridging (e.g. 8 + 4 and 4 + 8)	
Compensating	

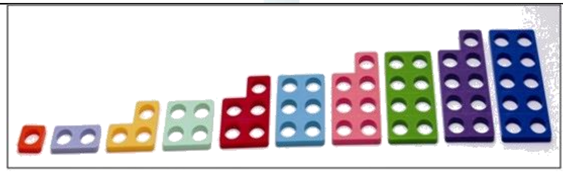
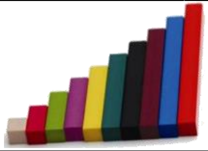
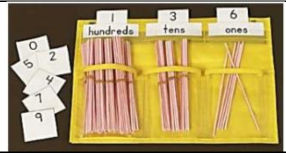
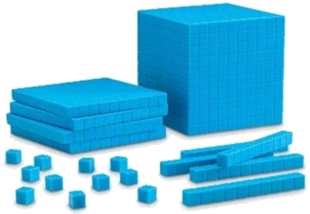
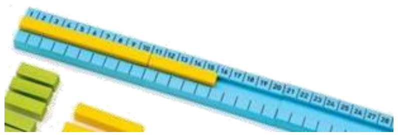
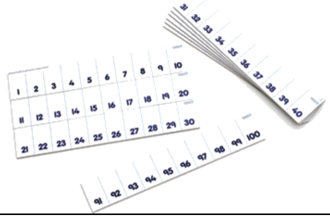
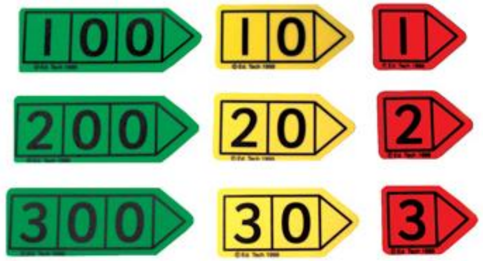
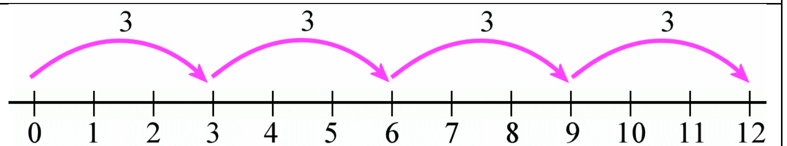
Multiplication and Division Facts

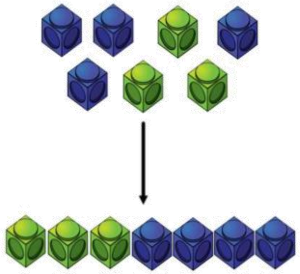
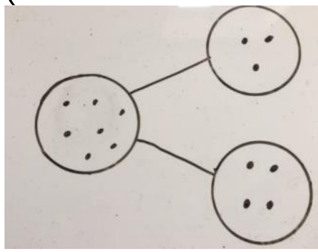
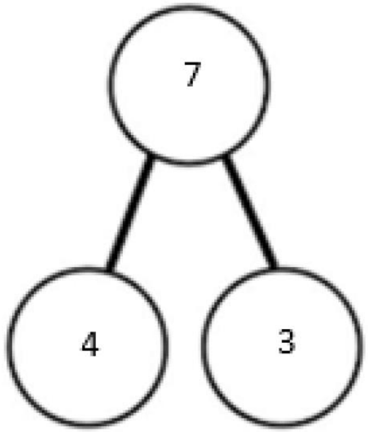
Year 2	recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables
Year 3	recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables
Year 4	recall multiplication and division facts for multiplication tables up to 12 × 12

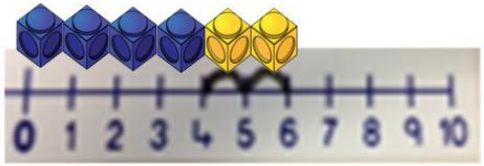
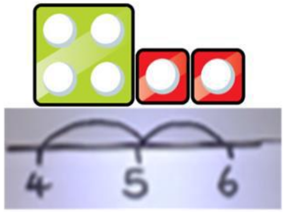
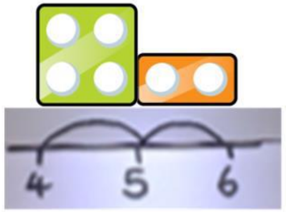
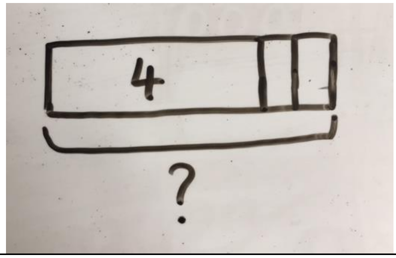
We learn the tables in this order:

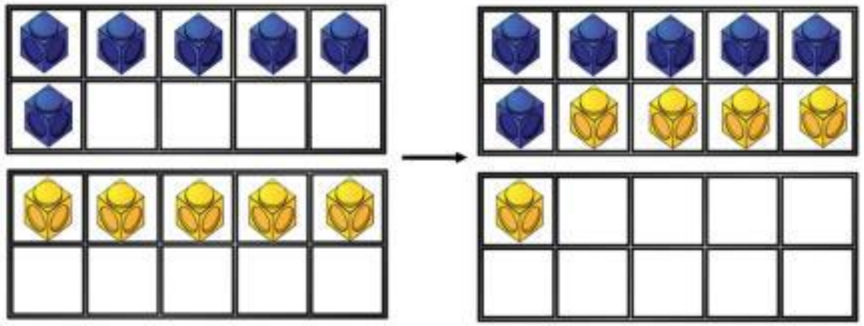
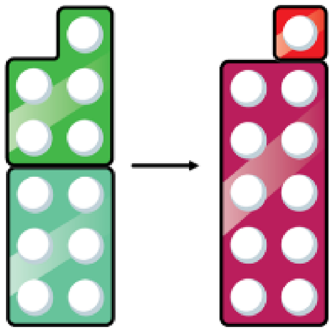
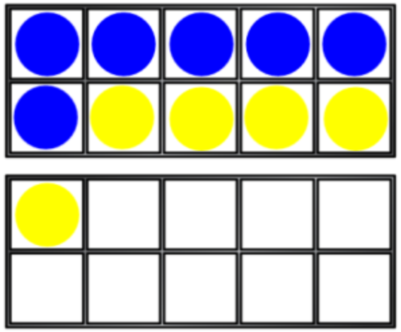
x10	x5	x2	x4	x8	x3	x6	x9	x7	x11	x12
Year 2			Year 3			Year 4				

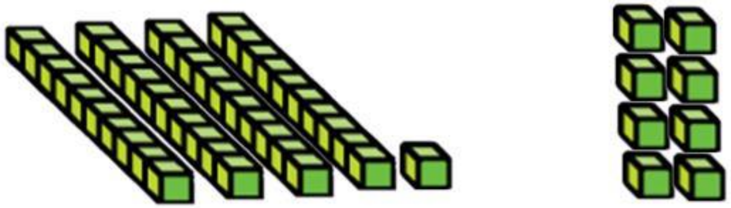
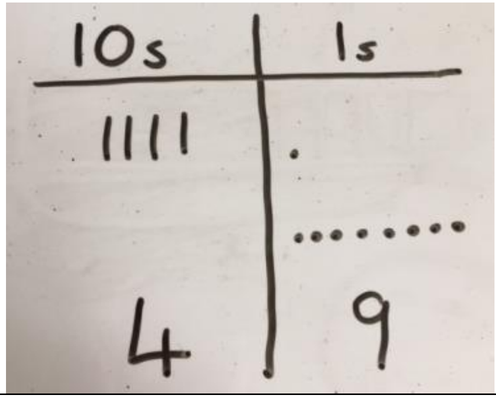
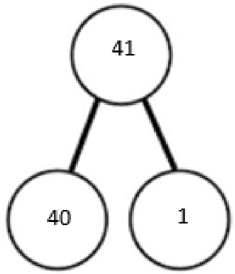
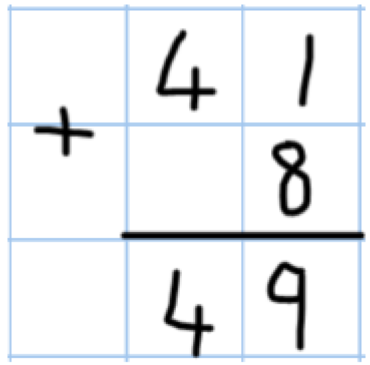
Representing Numbers with concrete materials

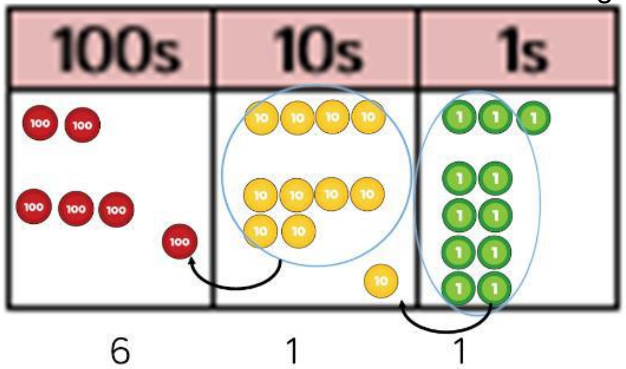
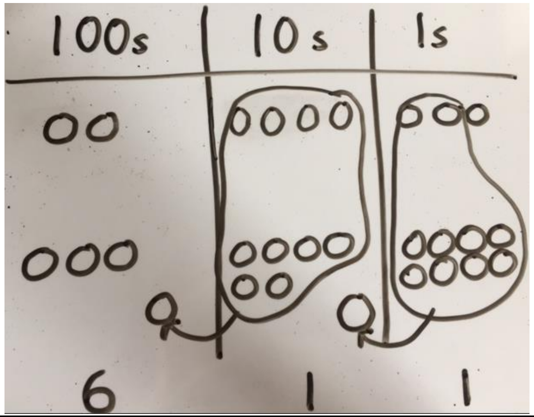
Objects		Counters	
Pegs		Numicon Shapes	
Number Rods (Cuisenaire)		Straws	
Place Value Equipment		Place Value Counters	
Number Tracks	 	Place Value Cards	
Bead Strings		Number Lines	

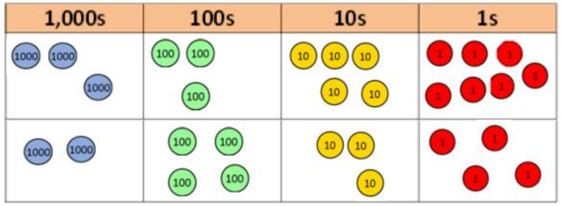
Addition	Concrete & Pictorial Representations	Written Recording
Reception Combining two parts to make a whole	 (use other resources too e.g. eggs, shells, teddy bears, cars).  Children to represent the cubes using dots or crosses. They could put each part on a part whole model too.	**4 + 3 = 7** **7 = 4 + 3** Four is a part, three is a part and the whole is seven. 

Addition	Concrete & Pictorial Representations	Written Recording
Year 1 Starting at the bigger number and counting on	Count on using cubes or Numicon.    A bar model which encourages the children to count on, rather than count all. 	**The abstract number line:** What is 2 more than 4? What is the sum of 2 and 4? What is the total of 4 and 2? 4 + 2 

Addition	Concrete & Pictorial Representations	Written Recording
Year 1 Exchanging to make 10	Exchange using ten frames and counters/cubes or using Numicon. 6 + 5   Children to draw the ten frame and counters/cubes. 	Children to develop an understanding of equality e.g. $6 + \square = 11$ $6 + 5 = 5 + \square$ $6 + 5 = \square + 4$

Addition	Concrete & Pictorial Representations	Written Recording
Year 2 Adding two or three single digits. Column layout – no exchanging.	**TO + O using base 10.** Continue to develop understanding of partitioning and place value.  41 + 8 Children to represent the base 10 e.g. lines for tens and dot/crosses for ones. 	**41 + 8** 1 + 8 = 9 40 + 9 = 49  

Addition	Concrete & Pictorial Representations	Written Recording
Year 3 Column layout- exchanging (up to 3 digits).	**Place Value Counters** Use of place value counters to add HTO + TO, HTO + HTO etc. When there are 10 ones in the 1s column- we exchange for 1 ten, when there are 10 tens in the 10s column- we exchange for 1 hundred.  Children to represent the counters in a place value chart, circling when they make an exchange.  	243 + 368 11 611

Addition	Concrete & Pictorial Representations	Written Recording
Year 4 Column layout- exchanging. (up to 4 digits)	**Place Value Counters** on Grids – Thousands, Hundreds, Tens and Units 	**Compact column addition** with larger numbers e.g. *5347 + 2286 + 1495* 5347 2286 + 1495 121 9128
Year 5 Column layout- exchanging (including decimals) .	**Place Value Counters** on Grids – including Tenths and Hundredths	**Compact column addition** to add pairs of 5-digit numbers Continue to use column addition to add towers of several larger numbers Use **compact addition** to add decimal numbers with up to 2 decimal places e.g. *15·68 + 27·86* 15·68 + 27·86 11·1 43·54

<table>
<tr><th>Subtraction</th><th>Concrete and Pictorial Representations</th><th>Written Recording</th></tr>
<tr><td>

Reception
Subtraction by taking away

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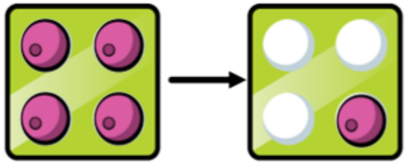
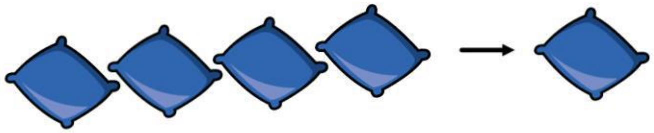
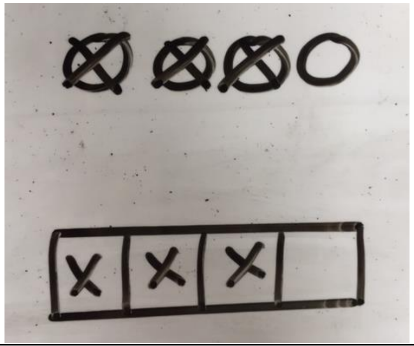
Physically taking away and removing objects from a whole (ten frames, Numicon, cubes and other items such as beanbags could be used).
4 – 3 = 1





Children to draw the concrete resources they are using and cross out the correct amount. The bar model can also be used.



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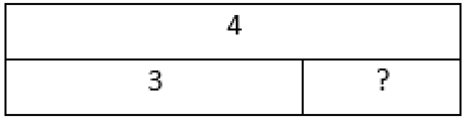
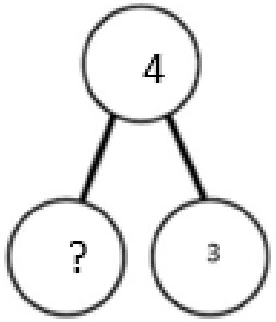
4- 3 = $\square$

$\square$ = 4 – 3





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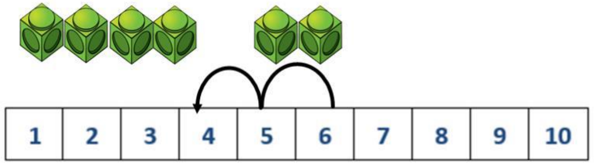
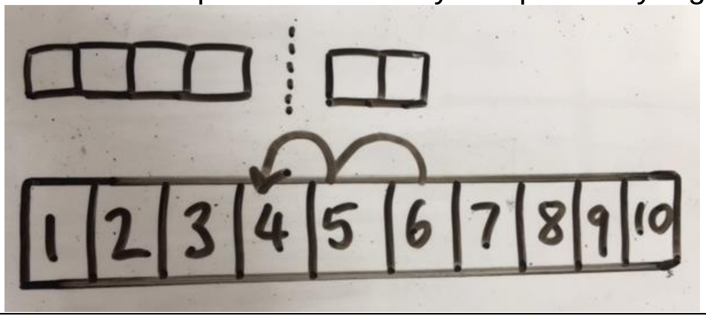
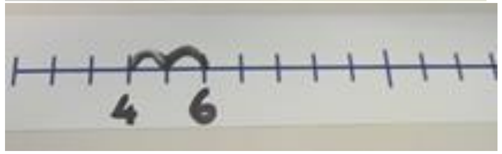
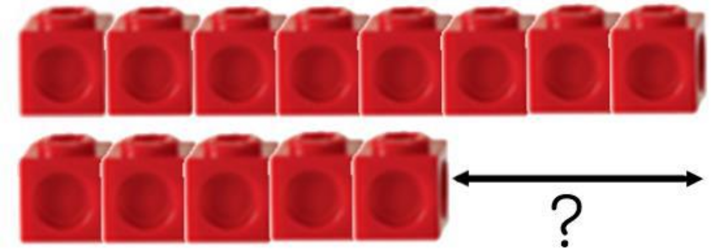
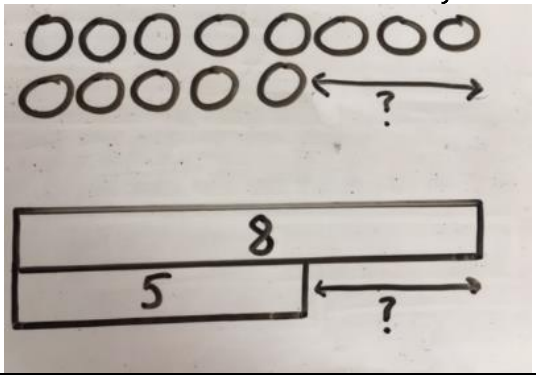
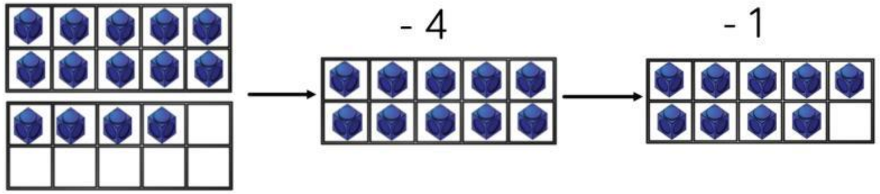
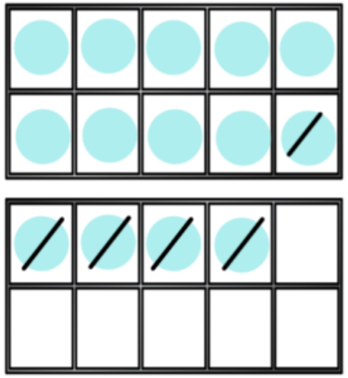
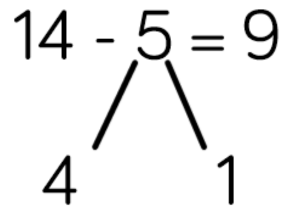
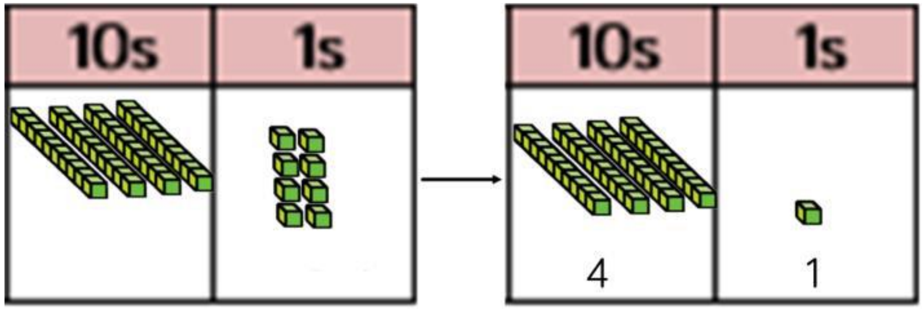
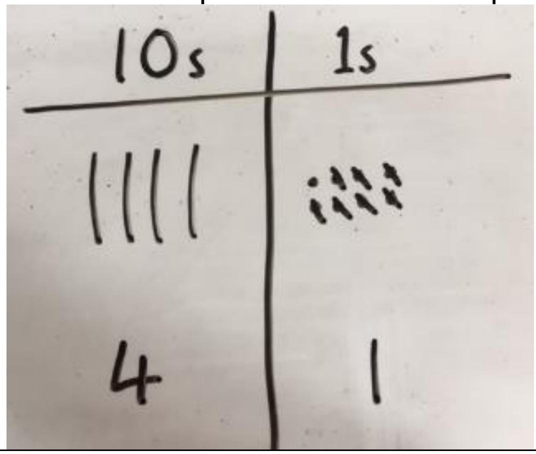
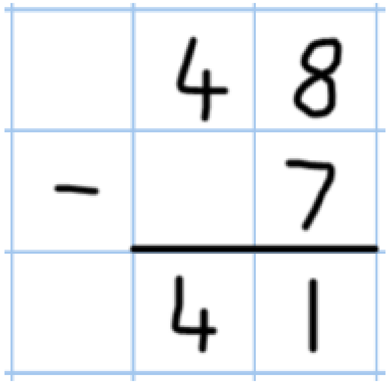
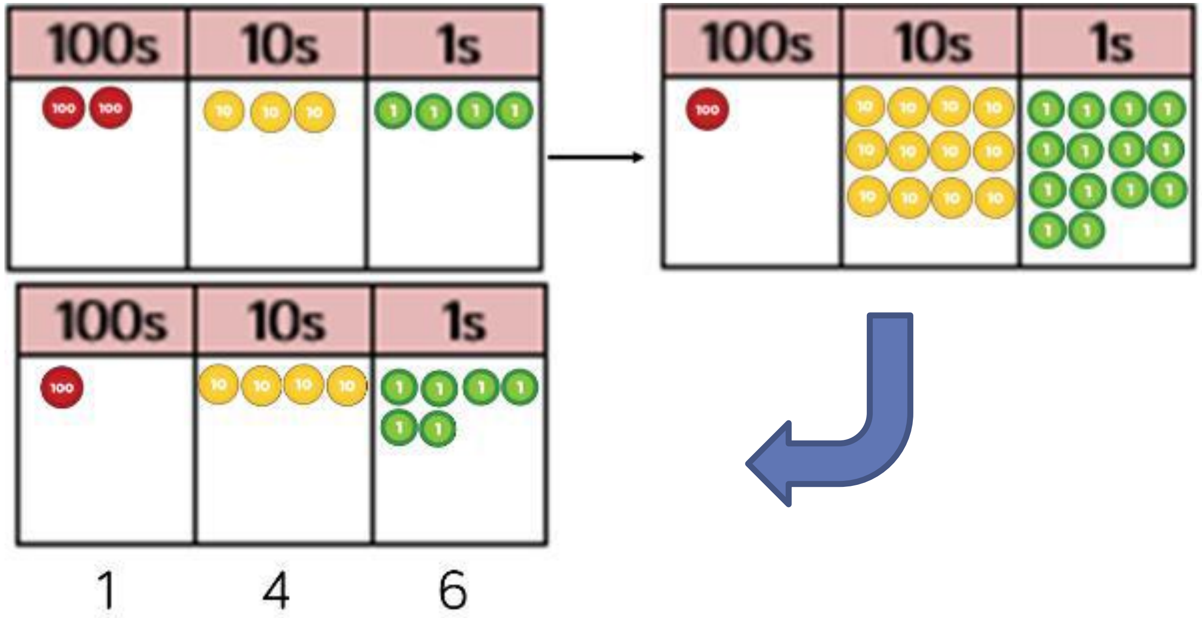
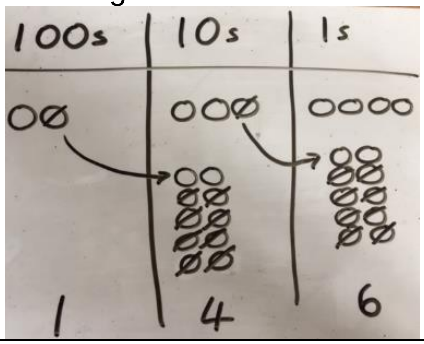
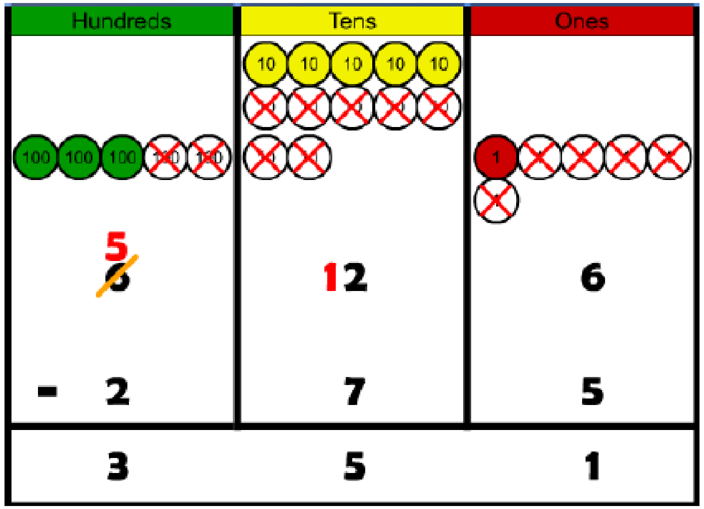
Subtraction	Concrete and Pictorial Representations	Written Recording
Reception / Year 1 Subtraction by counting back	Counting back (using number lines or number tracks) children start with 6 and count back 2. 6 – 2 = 4  Children to represent what they see pictorially e.g.  	Children to represent the calculation on a number line or number track and show their jumps. Encourage children in Year 1 to use an empty number line  

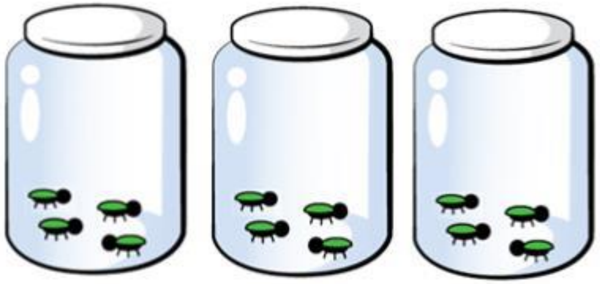
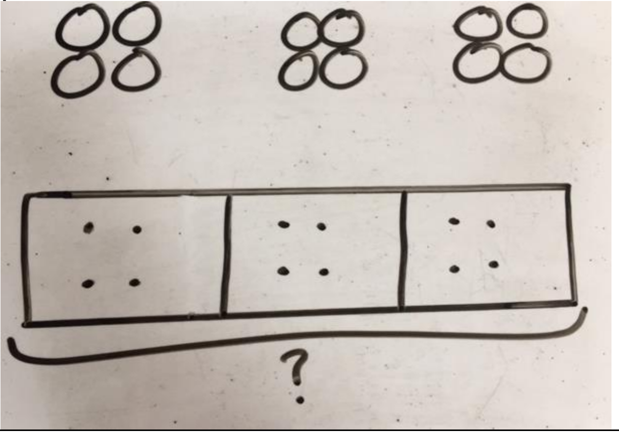
Subtraction	Concrete and Pictorial Representations	Written Recording
Year 1 Find the difference by	Finding the difference (using cubes, Numicon or Cuisenaire rods, other objects can also be used). Calculate the difference between 8 and 5.  Children to draw the cubes/other concrete objects which they have used or use the bar model to illustrate what they need to calculate.  	Find the difference between 8 and 5. 8 – 5, the difference is [] Children to explore why 9 - 6 = 8 – 5 = 7 – 4 have the same difference.

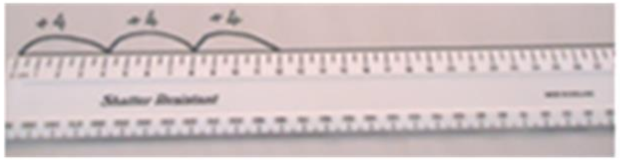
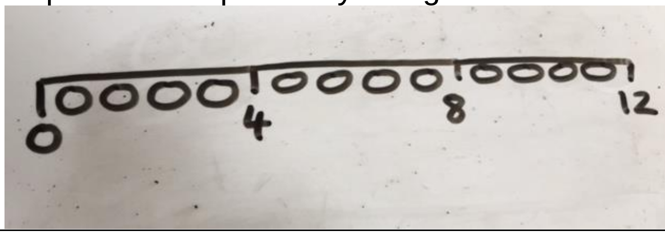
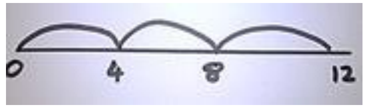
Subtraction	Concrete and Pictorial Representations	Written Recording
Year 1 Subtraction within 20	Making 10 using ten frames. 14 – 5  Children to represent the ten frame pictorially and discuss what they did to make 10. 	Children to show how they can make 10 by partitioning the subtrahend.  14 - 4 = 10 10 - 1 = 9

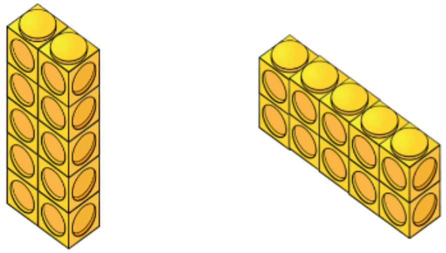
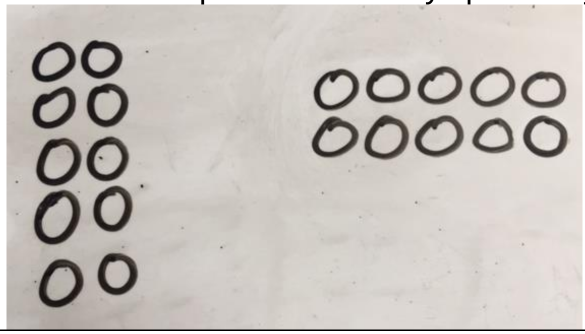
Subtraction	Concrete and Pictorial Representations	Written Recording
Year 2 Column layout – no exchanging	Using Base 10 apparatus to take away without exchanging 48-7  Children to represent the base 10 pictorially, crossing out  	Column layout or children could count back 7. 

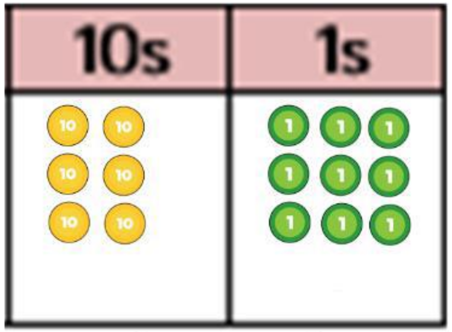
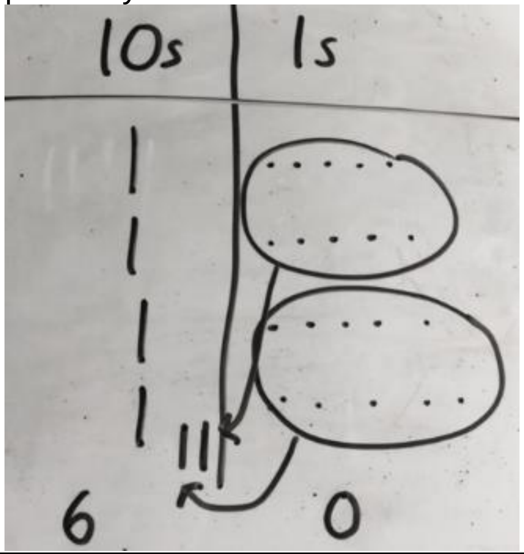
Subtraction	Concrete and Pictorial Representations	Written Recording
Year 3 Column layout with exchanging (up to 3 digits)	Using place value counters with exchanging (exchange) 234 – 88  Represent the place value counters pictorially; remembering to show what has been exchanged. 	Formal column layout. Children must understand what has happened when they have crossed out digits. $\begin{array}{r} 2\overset{2}{}\cancel{3}\overset{1}{}4 \\ -\ \ 88 \\ \hline 6 \\ \hline \end{array}$

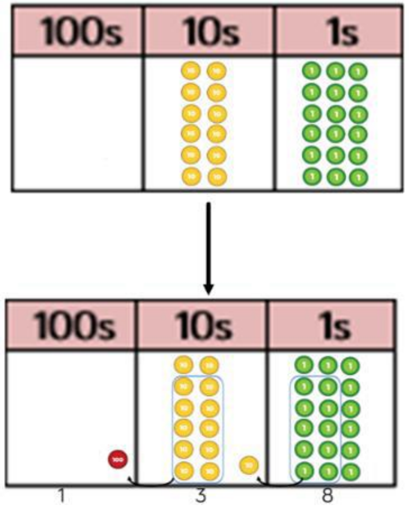
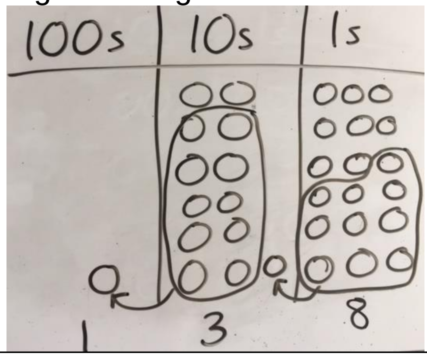
Subtraction	Concrete and Pictorial Representations	Written Recording
Year 4 Column subtraction with exchanging (up to 4 digits).	**Place Value Counters** on Grids – Thousands, Hundreds, Tens and Units. Children may need to exchange more than once. 	Develop **compact column subtraction with more than one exchange** e.g. *726 – 358* $\begin{array}{cccc} & 6 & 11 & 16 \\ & \cancel{7} & \cancel{2} & \cancel{6} \\ - & 3 & 5 & 8 \\ \hline & 3 & 6 & 8 \\ \hline \end{array}$
Year 5 Column subtraction with exchanging (2-place decimals)	**Place Value Counters for decimals** on Grids – including Tenths and Hundredths **Money** – Notes and Coins	**Compact column subtraction** for numbers with up to 5 digits e.g. *16 324 – 8516* $\begin{array}{cccccc} & 0 & 15 & 13 & 1 & 14 \\ & \cancel{1} & \cancel{6} & \cancel{3} & \cancel{2} & \cancel{4} \\ - & & 8 & 5 & 1 & 6 \\ \hline & & 7 & 8 & 0 & 8 \\ \hline \end{array}$ Subtract decimals with the same number of decimal places

Multiplication	Concrete and Pictorial Representations	Written Recording
Year 1 Recognising and making equal groups. Doubling Counting in multiples	Repeated grouping/repeated addition 3 × 4 (3 groups of 4) 4 + 4 + 4 There are 3 equal groups, with 4 in each group.   Children to represent the practical resources in a picture and use a bar model. 	$3 \times 4 = 12$ $4 + 4 + 4 = 12$

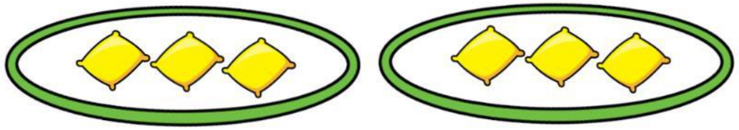
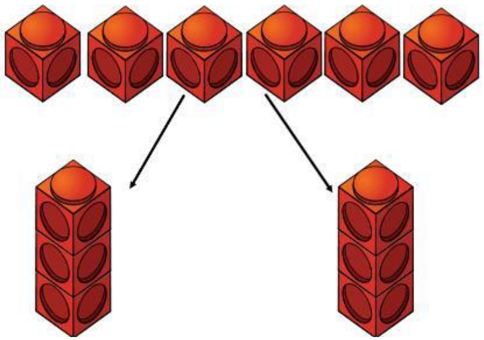
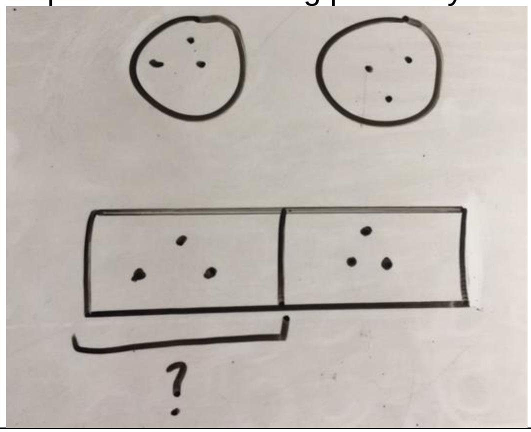
Multiplication	Concrete and Pictorial Representations	Written Recording
Year 2	Number lines to show repeated groups 3 x 4 (3 lots of 4)   Cuisenaire rods can be used too. Represent this pictorially alongside a number line e.g.: 	Abstract number line showing three jumps of four. $3 \times 4 = 12$ 

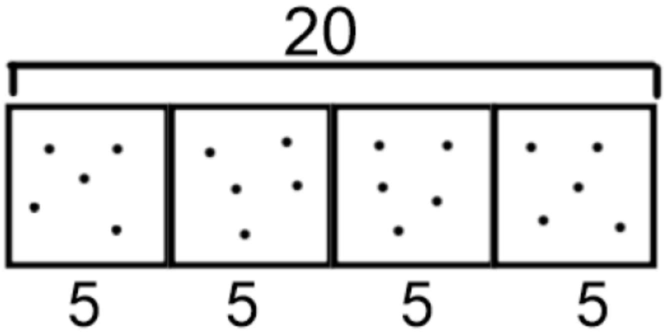
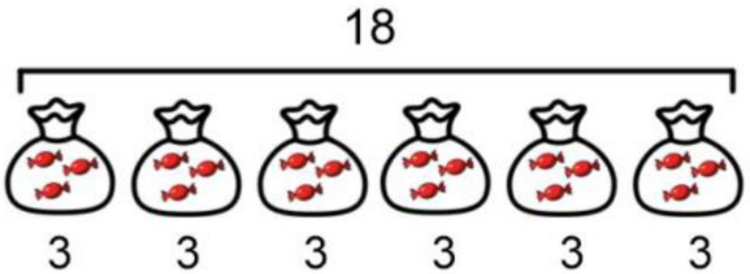
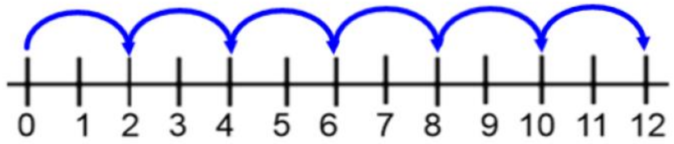
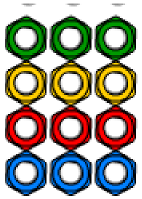
Multiplication	**Concrete and Pictorial Representations**	**Written Recording**
Year 2 Arrays- showing commutative multiplication	Use arrays to illustrate commutativity counters and other objects can also be used. $2 \times 5 = 5 \times 2$  2 lots of 5 5 lots of 2 Children to represent the arrays pictorially. 	Children to be able to use an array to write a range of calculations e.g. $10 = 2 \times 5$ $5 \times 2 = 10$ $2 + 2 + 2 + 2 + 2 = 10$ $10 = 5 + 5$

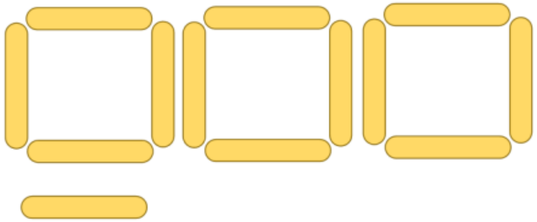
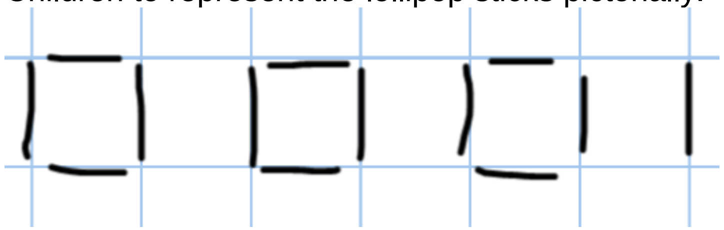
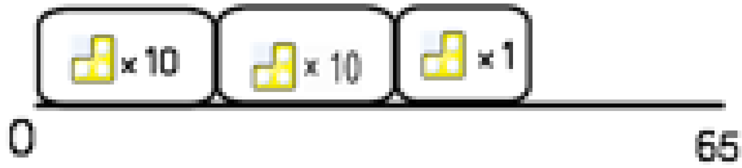
Multiplication	Concrete and Pictorial Representations	Written Recording
Year 3 Column layout to TU x U	Using place value counters (base 10 can also be used.) 3 × 23 = 3 lots of 23 = 3 lots of 20 and 3 lots of 3  6 9 Children to represent the concrete manipulatives pictorially. 	Children to record what it is they are doing to show understanding. 3×23 20 3 $3 \times 20 = 60$ $3 \times 3 = 9$ $60 + 9 = 69$ $\begin{array}{r} 23 \\ \times \;\; 3 \\ \hline 69 \\ \hline \end{array}$

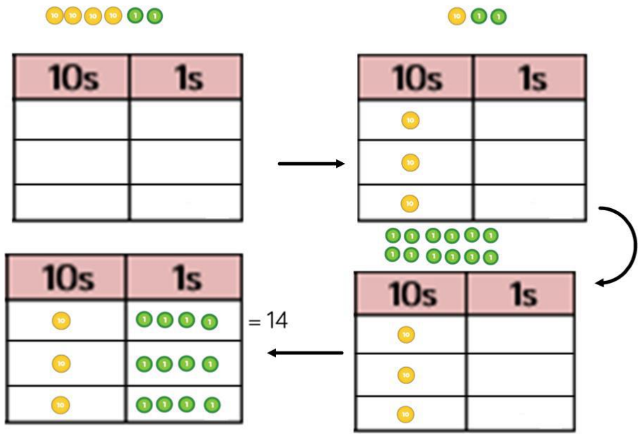
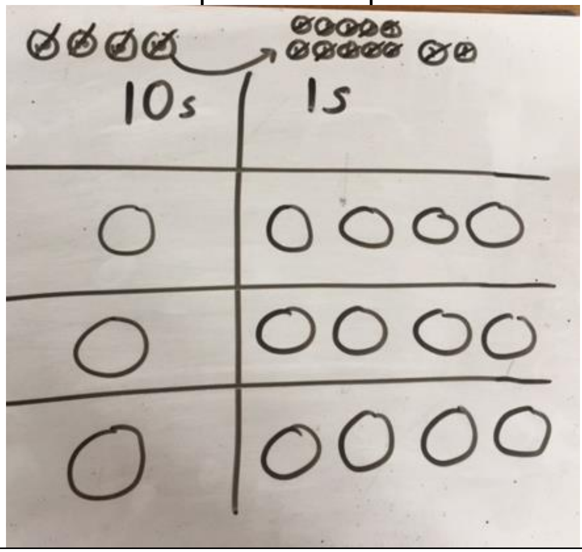
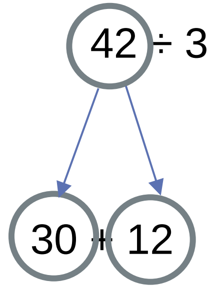
Multiplication	Concrete and Pictorial Representations	Written Recording
Year 4 Column multiplication (2 and 3 digit multiplied by 1 digit)	Using place value counters (with exchanging)  Children to represent the counters/base 10, pictorially e.g. the image below.  	Formal written method $6 \times 23 =$ $\begin{array}{r} 23 \\ \times \quad 6 \\ \hline 138 \\ \hline {\scriptstyle 1\ 1\ } \end{array}$ Note: exchanges written beneath the line.

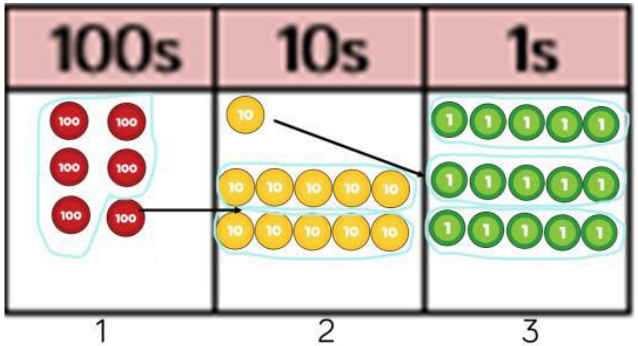
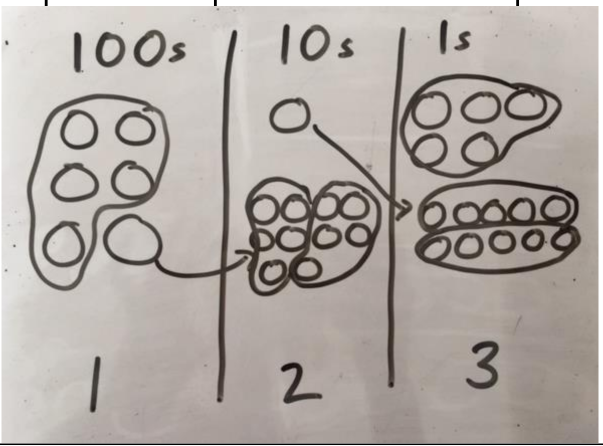
Multiplication	Concrete and Pictorial Representations	Written Recording
Year 5 Column multiplication	Abstract only but might need a repeat of year 4 first(up to 4 digit numbers multiplied by 1 or 2 digits)	**Long multiplication** of 2-, 3-and 4-digit numbers by 2-digit numbers e.g. *124 x 26* 1 2 4 × 2 6 7 4 4 1 2 2 4 8 0 3 2 2 4 1 1 Answer: 3224

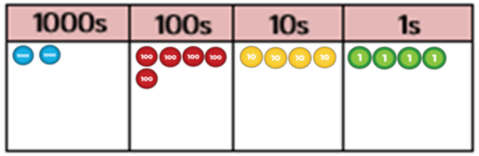
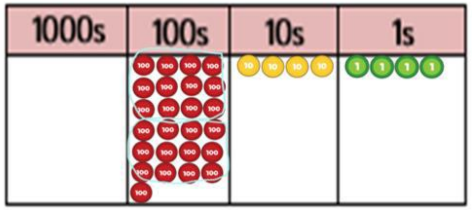
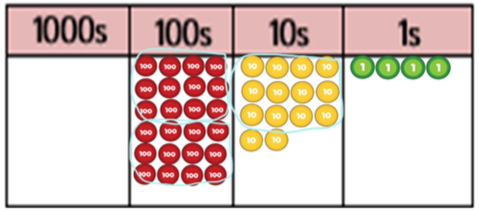
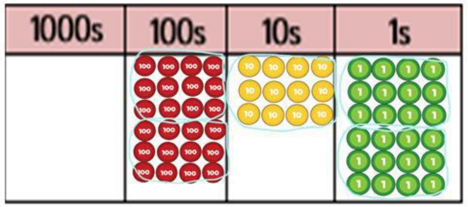
Division	Concrete and Pictorial Representations	Written Recording
Year 1 Sharing objects into groups Division as grouping	Sharing using a range of objects. 6 ÷ 2   Represent the sharing pictorially. 	6 ÷ 2 = 3 3 3

Division	Concrete and Pictorial Representations	Written Recording
Year 2 Division as sharing Repeated addition	**Sharing into equal groups** : e.g. 20 divided by 4 is 20 shared equally into 4 parts  **Grouping and counting** e.g. 18 divided by 3 is 18 split into equal groups of 3  	Abstract number line to represent the equal groups that have been added. 
Year 2 Division within arrays- linking to multiplication	Recognise the links between multiplication and division through use of arrays: 3 x 4 = 12 4 x 3 = 12 12 ÷ 4 = 3 12 ÷ 3 = 4 	

Division	Concrete and Pictorial Representations	Written Recording
Year 3 Division with a remainder	2d ÷ 1d with remainders using lollipop sticks. Cuisenaire rods, above a ruler can also be used. 13 ÷ 4 Use of lollipop sticks to form wholes- squares are made because we are dividing by 4.  There are 3 whole squares, with 1 left over. Children to represent the lollipop sticks pictorially.  There are 3 whole squares, with 1 left over.	65 divided by 3 equals 21 remainder 2 Children should be encouraged to use their times table facts; they could also represent repeated addition on a number line. ‘21 groups of 3, with 2 left over’ $65 \div 3 =$ 

Division	Concrete and Pictorial Representations	Written Recording
Year 3 2d divided by 1d	Sharing using place value counters (or base 10) 42 ÷ 3 = 14  Children to represent the place value counters pictorially. 	Children to be able to make sense of the place value counters and write calculations to show the process.  30 ÷ 3 = 10 12 ÷ 3 = 4 10 + 4 = 14

Division	Concrete and Pictorial Representations	Written Recording
Year 4 Short division (up to 3 digits by 1 digit- concrete and pictorial)	Short division using place value counters to group. 615 ÷ 5  1. Make 615 with place value counters. 2. How many groups of 5 hundreds can you make with 6 hundred counters? 3. Exchange 1 hundred for 10 tens. 4. How many groups of 5 tens can you make with 11 ten counters? 5. Exchange 1 ten for 10 ones. 6. How many groups of 5 ones can you make with 15 ones? Represent the place value counters pictorially. 	Children do the calculation using the short division scaffold. 123 5 | 6 ^{1}1 ^{1}5

<table>
<tr><th>Division</th><th>Concrete and Pictorial Representations</th><th>Written Recording</th></tr>
<tr><td>Year 5
Short division
(up to 4 digits by a 1 digit number including remainders)</td><td></td><td>Short division of 3- and 4-digit numbers by 1-digit numbers
e.g. 139 ÷ 3f
4 6 r 1
3 ⟌ 1 3 ¹9
Give remainders as whole numbers or as fractions</td></tr>
<tr><td>Year 6
Long division (up to 4 digits by a 2 digit number)</td><td colspan="2">Long division using place value counters
2544 ÷ 12

We can't group 2 thousands into groups of 12 so will exchange them.

We can group 24 hundreds into groups of 12 which leaves with 1 hundred.
$\begin{array}{r} 02 \\ 12\overline{)2544} \\ 24 \\ \hline 1 \end{array}$

After exchanging the hundred, we have 14 tens. We can group 12 tens into a group of 12, which leaves 2 tens.
$\begin{array}{r} 021 \\ 12\overline{)2544} \\ 24 \\ \hline 14 \\ 12 \\ \hline 2 \end{array}$

After exchanging the 2 tens, we have 24 ones. We can group 24 ones into 2 group of 12, which leaves no remainder.
$\begin{array}{r} 0212 \\ 12\overline{)2544} \\ 24 \\ \hline 14 \\ 12 \\ \hline 24 \\ 24 \\ \hline 0 \end{array}$
Children should exchange into the tenths and hundredths column too</td></tr>
</table>